Conceptualizing and Modeling Contextual Effects in Longitudinal Studies

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Context

Modeling Contextual Effects in Longitudinal Studies

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Context

• The circumstances in which an event occurs; a setting.
  — The set of features that influences the performance or the outcome of a process
• The conditions that are relevant to an event, fact, etc.
  — From contextus a putting together
  — From contexere to interweave, braid
  — circumstances, times, conditions, situation, ambience, frame of reference, background, framework, relation, connection
• Ecology
  — The relationship between organisms and their environment.
Figure 2.5. Overlapping Ecologies of Human Development
Figure 2.6. Bronfrenbrenner’s hierarchy of the Social Ecology
Figure 2.7. Widaman’s hierarchy of the Physical Ecology
Figure 2.8. Little’s hierarchy of the Personal Ecology
Contexts as

• **Direct effects**
  – Varies at the level of the individual and influences the individual directly

• **Indirect (mediated) effects**
  – Varies at the level of the individual and influences the individual through its effect on an intervening variable

• **Mediating effects**
  – Distal context influences proximal context which influences the individual

• **Moderating effects**
  – Interactive influences that change the strength of any of the above effects
    – Discrete vs. Continuous

• **Hierarchically nested effects**
  – Larger units of context that can have direct, indirect, mediating, or moderating effects.
Types of Variables

**Table 2.5. A simple taxonomy of the types of variables and their role in an analysis**

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<tr>
<th>&quot;Covariates&quot;</th>
<th>&quot;Predictors&quot;</th>
<th>&quot;Outcomes&quot;</th>
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These variables are used as statistical controls to account for their influence on other variables so that the influence among the other focal variables can be seen clearly. These variables are often seen as potential confounding influences that may create or mask a hypothesized relationship.

These variables are treated as the focal variables that are the primary determining influence on an outcome variable. They reflect the hypothesized process or mechanism that gives rise to the outcome variables.

These variables are also focal variables but reflect the outcome of interest. They are the end result of the predicted relationships that one tests after controlling for potential confounds.

Note. These categorizations are simplified and the precise technical meaning varies across disciplines and researchers.
• We should measure persons and contexts well
  ─ Measures should be appropriate for the construct
    • Contexts should be quantified (borrow from sociology, for example)
    • Developmental measures should address change
      ─ The tragic legacy of test-retest reliability
  ─ Measures and analyses should not be haphazard
    • Avoid: “Hey, this new method is cool, let’s try it on this data?”
    • Question -> Measurement -> Statistical Model
    • Avoid short forms of existing scales (use intentionally missing design)
      ─ (‘allure of the bloated specific’ idea)
    • Develop or modify to make sure the measurement tool is right
    • Take time to refine and pilot measures (even well-established ones).

"Whatever exists at all exists in some amount. To know it thoroughly involves knowing its quantity as well as its quality"

- E. L. Thorndike (1918)
Context of Measurement

• Homotypic vs. heterotypic expressions across ages
  — e.g., Aggression

• Surface-structure vs. deep-structure of behavior
  — e.g., helping as resource-directed behavior

• Typological (subgroups) differences
  — Identification issues and procedures
    • Muthen’s m-Plus, Nagin’s Proc Traj, Bergman’s Sleipner

• n-adic (dyadic, triadic, etc.) overlay on all of the various modeling approaches
  — e.g., SRM, APIM, Siena
Context of Change

- Interindividual differences vs. Intraindividual differences
  - Ergodicity conundrum
- Associations (within and between time)
  - Covariances and Correlations vs. Regressions
  - Direct and Indirect effects
    - Auto-regressive vs. Cross-lagged
      - 1st-order vs. 2nd-order
    - Linear vs. non-linear
- Means and Variances
- Mediation vs. Moderation vs. Additive Effects
- $B = f(\text{age})$ vs. $\Delta = f(\text{time})$
Context of Growth

• Most growth trends are locally linear
  — May have insufficient range to model nonlinear trends
    • Shouldn’t blind us to the nonlinearities that will likely exist

• Most growth trends are globally nonlinear
  — Remember this when:
    • Invoking theory
    • Describing trends in data
    • Drawing conclusions and implications
  — Design studies to capture appropriate forms of nonlinearity
    • Quadratic or cubic forms
      — Measure well before and well after the ‘bend’
      — Use optimal design logic when planning measurements
    • Piecewise linear (or nonlinear) models
Context of Time

• Age in years, months, days.

• Experiential time: Amount of time something is experienced
  — Years of schooling, length of relationship, amount of practice
  — Calibrate on beginning of event, measure time experienced

• Episodic time: Time to and from onset of a life event
  — Toilet trained, driver license, puberty, birth of child, retirement
  — Early onset, on-time, late onset: used to classify or calibrate.
  — Time since onset or time from normative or expected occurrence.

• Measurement Intervals (rate and span)
  — How fast is the developmental process?
  — Intervals must be equal to or less than expected processes of change
  — Measurement occasions must span the expected period of change
  — Cyclical processes
    • E.g., schooling studies at yearly intervals vs. half-year intervals
# Transforming to Episodic Time

## Data Collection Wave Crossed with Episode Occurrence

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Context of Validity in Longitudinal Work

• Threats to Validity
  — Maturation
    • In pre-post experiment effects may be due to maturation not the treatment
    • For longitudinal studies, maturation is the focus.
  — Regression to the mean
    • Only applicable with measurement error
  — Instrumentation effects (factorial invariance)
    • Testable
  — Test-retest effects
    • Use intentionally missing data designs to estimate and mitigate
  — Selection Effects
    • Sample Selectivity vs. Selective Attrition, both are mitigated by MAR assumption and modern missing data imputation.

• Age, Cohort, and Time of Measurement are confounded
  — Sequential designs attempt to unconfound these.
The Sequential Designs

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Cohort-Sequential

Time-Sequential

Cross-Sequential
# What’s Confounded?

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<th>Design</th>
<th>Independent Variables</th>
<th>Confounded Effect</th>
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<td>Cohort &amp; Time</td>
<td>Cohort x Time Interaction is confounded with Age</td>
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Context and Temporal Design

• Changes (and causes) take time to unfold.
• The ability to detect an effect depends on the measurement interval.
• The ability to model the shape of the effect requires adequate sampling of time intervals.
• The ability to model the optimal effect requires knowing the shape in order to pick the optimal (peak) interval.
• Lag within Occasion: Lag as Moderator analyses (Selig, Preacher, & Little, under review)
Types of Change Effects

A) Step Functional

B) Linear Increase

C) Cumulative Ogive

D) Quick & Dissapates

E) Uniform Raise & Fall

F) Quick & Tapers
Three General Techniques of Longitudinal Modeling

- **Panel models**
  - Focus on stability / prediction of interindividual differences (rank order), at expense of information about means
  - Commonly used & well understood
  - Can use with 2 time points. Is more complex/informative with 3 or more

- **Growth curve models**
  - Focus on intraindividual stability / change, as well as interindividual variability in intraindividual growth
  - Need at least 3 time points (for nonsaturated linear model). Easy to accommodate more or various numbers across people
  - Typically says nothing about direction of influence

- **P-techniques**
  - Focus on moment-to-moment covariation within an individual
  - Need at least 1 person, but many time points.
  - Can combine results across people to drawn nomothetic generalizations
  - Introduction of lag (dynamic P) allows conclusions of temporal primacy
Mediator vs. Moderator

• Mediator is the middle-person, letter carrier, delivery agent
  —X predicts M, X predicts Y, and M predicts Y
  —M to Y accounts for X to Y via (X-to-M-to-Y)
  —Mediation still occurs when X to Y is necessary, and X-to-M-to-Y is still significant.

• Moderator is an interaction!
  —It Depends. If A then B, if not A then C
  —Depending on B, A to C changes in strength

• Mediator is the carrier, Moderator is the changer
Pieces of the Mediation Puzzle

$C$ (the regression weight or total effect of $X$ on $Y$)

$a$ (the semi-partial regression weight or direct effect of $X$ on $Y$, controlling for $M$ and the $X$ to $M$ indirect path to $Y$ via $M$)

$b$
Depicting Moderation in Path Diagram

Note: Although this is how we draw the model in our diagram, the actual model fit is different (involves predicting Y from X, Moderator, and X-M interaction).
Four General Approaches to Mediation

• Baron & Kenny’s Causal Steps approach
  – Do a series of regressions and determine if $C’$ is $< C$.
  – No good test of significance of the change and it’s the wrong parameter!

• Product of Coefficients approach (Sobel test)
  – Determine if $ab$ is significant.
  – Relies on normal theory standard errors

• SEM estimation of indirect effect
  – Determine if $ab$ is significant
  – Uses ML estimation to determine effect; use chi-squared difference test for assessing significance

• Bootstrap approach
  – Determine if $ab$ is significant
  – Relies on resampling to determine the appropriate standard error to test for significance
Advice for testing mediation

• Do pilot work to detect the time interval that must elapse for X to have an effect on M and for M to have an effect on Y.
  • Waves of assessment should be separated by these empirically determined time intervals.

• Use latent variables; measurement error can wreak havoc on indirect effects.

• Specify the developmental time frame over which the mediation supposedly unfolds.
  • Represent this time period in its entirety.

• Use the overall indirect effect, not just time-specific indirect effects, to represent the mediation effect of interest.
Advice for testing mediation

• Ensure that the measurement (CFA) model fits well.
  • If it does not, we cannot trust anything else.
• Test *longitudinal factorial invariance*
  • If the latent variables do not have the same interpretation over time, there is little point in proceeding.
• Test equilibrium with the CFA model
  • Constrain latent variable variances and covariances to be equal over time and evaluate loss of fit (or not).
• Include lagged $X \rightarrow Y$ effects and correlated residuals at each occasion (not pictured).
• Permit lagged residuals to intercorrelate to account for shared method variance.
If the assumption of stationarity can be made (and this is a big “if”), the minimum design for mediation is:

Advantages:

1. Explicitly models change in both Y and M
2. Permits experimental manipulation of both X and M.
Interrelationships of Positive Affect, Possession of Ability, and Unknown Causes

Agency (1) -> Unknown causes (2) -> Positive Affect (3)

1* -> 1* -> 1*
Interrelationships of Positive Affect, Possession of Ability, and Unknown Causes

Note that the slope varies from .23 to -.45 as a function of Unknown Causes.
Orthogonalizing: Step 1

Interact = MainEffect1*MainEffect2;
AgeSQ = age*age;

Proc REG data=work;
Model Interact = MainEffect1 MainEffect2;
Output out=work  r = _Interact_;

Proc REG data=work;
Model AgeSQ = age;
Output out=work  r = _AgeSQ_;

Note that r is the residual which equals:
  r = observed y – predicted y
Use Residuals in Analyses: Step 2

Proc REG data=work;
   Model DV = MainEffect1 MainEffect2 _Interact_;

Proc REG data=work;
   Model DV = age _AgeSQ_;

Note _Interact_ is uncorrelated with the two main effects and _AgeSQ_ is uncorrelated with age. This was done in step 1
Extension to SEM: Step 1

- Compute all possible interactions among indicators
  - \( V_{14} = V_1 \times V_4 \)
  - \( V_{15} = V_1 \times V_5 \)
  - \( V_{16} = V_1 \times V_6 \)
  - \( V_{24} = V_2 \times V_4 \)
  - \( V_{25} = V_2 \times V_5 \)
  - \( V_{26} = V_2 \times V_6 \)
  - \( V_{34} = V_3 \times V_4 \)
  - \( V_{35} = V_3 \times V_5 \)
  - \( V_{36} = V_3 \times V_6 \)
Extension to SEM: Step 2

- Orthogonalize all interaction terms with respect to each main effect indicator
- For example:

  Proc REG data=work;
  Model V14 = V1 V2 V3 V4 V5 V6;
  Output out=work r = _V14_;
Interrelationships of Positive Affect, Possession of Ability, and Unknown Causes

- **Agency** (1)
  - 1
  - 2
  - 3

- **Unknown causes** (2)
  - 1
  - 4
  - 5
  - 6

- **Positive Affect** (3)
  - 1

- **Interaction** (4)
  - 1
  - 4
  - 5
  - 6
  - 1*4
  - 1*5
  - 1*6
  - 2*4
  - 2*5
  - 2*6
  - 3*4
  - 3*5
  - 3*6
## The Residual Matrix

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Interrelationships of Positive Affect, Possession of Ability, and Unknown Causes

-0.28 (Z=8.27)

-0.07 (Z=2.09)

0.07 (Z=2.09)

0.29 (Z=8.28)

1*

1*

1*

1*

(13.5. Moderation)
Katherine Maysn’s Example of Nested Data Structures
Negative Individual, Positive Group
Positive Individual, Negative Group
No Individual, Positive Group
No Group, Mixed Individual
Multilevel Structures

- Observations at one level are nested within observations at another and so on.

- Number of levels theoretically limitless, bounded by practicality (and software).
  - Random sampling at each level.

- Multilevel vs. multiple-group structures

- Lowest level observations are not independent—possible biases in parameter estimates, standard errors, and test of model fit.
  - Goal is to model both within- and between-cluster relationship.

- Examples:
  - Students within classrooms
  - Times of measurement within persons
• Info and Registration links for our annual Summer Institutes
  - (SEM, Longitudinal SEM, Multilevel, Categorical, Social Network with Siena, Statistical Analysis with R, Meta-analysis, IRT).

• KUant Guides and other online resources
Recommended readings

  - Introduction to special issue, but first part identifies basic issues in longitudinal modeling and then points you to the innovations covered in the special issue.
  - Provides a broad summary of the three classes of techniques
  - Introduction to our book that points you to a lot of really great chapters covering many of these issues in detail
Recommended readings

  - Convincing argument for testing mediation over time rather than concurrently

  - The first half of this chapter describes testing mediation in SEM and the second half covers moderation

  - If the first paper wasn’t convincing, then this one surely should get the point across